General Directions For Students :Whatever be the notes provided , everything must be copied in the maths copy and then do the homework in the same copy

Chapter 5: Exercise 5.7

Topic : Derivatives of Inverse Trigonometric Functions

Differentiation Formulas For Inverse Trigonometric Function:

1. $\frac{d}{dx}\left(\sin^{-1}x\right)=\frac{1}{\sqrt{1-x^2}},|x|<1$

2. $\frac{d}{dx}\left(\cos^{-1}x\right)=-\frac{1}{\sqrt{1-x^2}},|x|<1$

3. $\frac{d}{dx}\left(\tan^{-1}x\right)=\frac{1}{1+x^2},x\varepsilon R$

4. $\frac{d}{dx}\left(\cot^{-1}x\right)=-\frac{1}{1+x^2},x\varepsilon R$

4. $\frac{d}{dx}\left(\sec^{-1}x\right)=\frac{1}{|x|\sqrt{x^2-1}},|x|>1$

5. $\frac{d}{dx}\left(co\sec^{-1}x\right)=-\frac{1}{|x|\sqrt{x^2-1}},|x|>1$

Some useful suggestion given below:

If the function contains an expression of the form:

i. $a^2 - x^2$, put x =a sin t or a cos t
ii. $a^2 + x^2$, put x =a tan t or a cot t
iii. x^2-a^2 , put $x=a\sec t$ or a cosec t
iv. $\sqrt{\frac{a-x}{a+x}}$ or $\sqrt{\frac{a+x}{a-x}}$ put x =a cos t
v. $a\cos x\pm b\sin x$, put a =r cos α and b = r sin α , r >0

Exercise 5.7, Q2.i) Find the derivative of $\tan^{-1}(x\sqrt{x})$.

solution. $\frac{d}{d(x\sqrt{x})}(\tan^{-1}(x\sqrt{x})).\frac{d}{dx}(x\sqrt{x})$

$=\frac{1}{\left(1+\left(x\sqrt{x}\right)^2\right)}.\frac{3\sqrt{x}}{2}$

$=\frac{1}{\left(1+x^3\right)}.\frac{3\sqrt{x}}{2}$

$=\frac{3\sqrt{x}}{2\left(1+x^3\right)}.$

Q.3.ii) Find the derivative of $\tan^{-1}(\cot x)$.

Let y= $\tan^{-1}(\cot x)$

y = $\tan^{-1}\left(\tan\left(\frac{\pi}{2}-x\right)\right)$

$\Rightarrow$ y = $\frac{\pi}{2}-x$

$\Rightarrow \frac{dy}{dx}=0-1$

$\Rightarrow \frac{dy}{dx}=-1$

Q.4ii) $y=\tan^{-1}(x)+\cot^{-1}\left(\frac{1}{x}\right)$, *find* $\frac{dy}{dx}$,x > 0

$y=\tan^{-1}(x)+\tan^{-1}(x),$ $\because \cot^{-1}\frac{1}{x}=\tan^{-1}x,$ x > 0

$\Rightarrow y=2\tan^{-1}x$

$\Rightarrow \frac{dy}{dx}=2\frac{d}{dx}(\tan^{-1}x)$

$\Rightarrow \frac{dy}{dx}=2\left(\frac{1}{1+x^2}\right)$

$\Rightarrow \frac{dy}{dx}=\frac{2}{1+x^2}$

Q.7.i) Differentiate $\cos^{-1}\left(\frac{1-x}{1+x}\right)$

Let y= $\cos^{-1}\left(\frac{1-x}{1+x}\right)$

let $\sqrt{x}=\tan t$

then y= $\cos^{-1}\left(\frac{1-\tan^2 t}{1+\tan^2 t}\right)=\cos^{-1}(\cos 2t)=2t$

$\Rightarrow$ y=2t= $2\tan^{-1}\sqrt{x}$

$$\Rightarrow \frac{dy}{dx} = 2\left(\frac{1}{1+\left(\sqrt{x}\right)^2}\right)\frac{d}{dx}\left(\sqrt{x}\right)$$

$$\Rightarrow \frac{dy}{dx} = \left(\frac{2}{1+x}\right)\frac{1}{2\sqrt{x}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{x}\left(1+x\right)}$$

Q.6.ii)Differentiate $x\cos^{-1}\sqrt{x}$:

Let y= $x\cos^{-1}\sqrt{x}$

$$\Rightarrow \frac{dy}{dx} = x\frac{d}{dx}(\cos^{-1}\sqrt{x}) + \cos^{-1}\sqrt{x}\,\frac{d}{dx}(x)$$

$$= x\left(-\frac{1}{\sqrt{1-x}}\right)\frac{d}{dx}(\sqrt{x}) + \cos^{-1}\sqrt{x}$$

$$= \left(-\frac{x}{\sqrt{1-x}}\right)\left(\frac{1}{2\sqrt{x}}\right) + \cos^{-1}\sqrt{x}$$

$$= -\frac{\sqrt{x}}{2\sqrt{1-x}} + \cos^{-1}\sqrt{x}$$

Q.9.iv) Differentiate $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$.

Let y = $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right) = \tan^{-1}\left(\frac{\sin\left(\frac{\pi}{2}-x\right)}{1+\cos\left(\frac{\pi}{2}-x\right)}\right) = \tan^{-1}\left(\frac{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right)$

$$\Rightarrow y = \tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right) = \frac{\pi}{4}-\frac{x}{2}$$

$$\frac{dy}{dx} = -\frac{1}{2}$$

Homework:

Exercise 5.7

Q2.vi), Q.3.i),Q.5.ii)Q.6iii), Q.7.iii),Q8.ii),Q.9.ii),Q.10.ii),Q11.ii),Q.14ii),Q. 16.ii)Q18.iv)